UNIVERSITATEA TEHNICA DE CONSTRUCTII BUCURESTI

CATEDRA DE MATEMATICA SI INFORMATICA

ABSTRACTS BOOK

A X-a Sesiune de Comunicări a
Catedrei de Matematică şi Informatică din
Universitatea Tehnică de Construcţii Bucureşti

23 Mai 2009

A X-a Sesiune de Comunicări a Catedrei de Matematică şi Informatică din Universitatea Tehnică de Construcţii Bucureşti - 23 Mai 2009

Stimaţi Colegi,

A X-a Sesiune de Comunicări a Catedrei de Matematică şi Informatică din Universitatea Tehnică de Construcţii Bucureşti se va desfăşura la sediul din B-dul Lacul Tei, Nr. 124 al Universităţii Tehnice de Construcţii Bucureşti în ziua de 23 Mai 2009. Se vor organiza secţiuni pe următoarele domenii:

- Analiză matematică, Analiză funcţională, Analiză numerică,
- Algebră, Geometrie,
- Ecuaţii diferenţiale, Ecuaţii cu derivate parţiale, Mecanică,
- Probabilităţi, Cercetări operaţionale, Statistică matematică,
- Informatică, Aplicaţii matematice în ştiinţe inginereşti, Utilizarea unor programe de calcul în cercetare şi predarea matematicii.

Comitetul de organizare:

- Prof. dr. Gavriil Păltineanu,
  Şeful Catedrei de Matematică şi Informatică din UTCB,
- Prof. dr. Ghiocel Groza,
- Conf. dr. Emil Popescu,
- Conf. dr. Ion Mierluş-Mazilu,
- Lector dr. Narcisa Teodorescu.

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A COMPACTNESS LEMMA IN THE SET $M_b(B)$ OF BOUNDED MEASURES ON (X,B)

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Abstract

We prove a compacteness criterion on $\mathrm{M_b}(\mathrm{B})$ and we improuve the famous Dunford-Pettis result.

ON A DENJOY-BOURBAKI TYPE INEQUALITY AND ITS APPLICATIONS

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Abstract

We prove an inequality of the type $f(x)-f(a)\le v_f(x)+\int_a^x f'(t)dt$, $\forall x\in[a,b]$ where $f:[a,b]\to R$ is derivable almost everywhere with respect to the Lebesgue measure. As consequence of this relation we deduce the classical Denjoy-Bourbaki inequality and a necessary and sufficient condition for the equality $f(x)=f(a)+\int_a^x \varphi(t)dt$, $\forall x\in[a,b]$ where φ is a Lebesgue integrable function on [a,b].

AXIAL TAYLOR-COUETTE FLOW IN AN ANNULUS DUE TO A TIME-DEPENDENT SHEAR

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Abstract

This paper deals with an important problem in physics and engineering, namely with the Taylor-Couette flow formation in a generalized Oldroyd-B fluid filling the annular region between two infinitely long coaxial cylinders, due to a axial time-dependent shear applied on

the surface of the inner cylinder. The obtained solution is presented as the sum of the corresponding Newtonian solution and the non-Newtonian contribution. This solution was been specialized to give the solution for generalized second grade or Maxwell fluids as well as the solution for ordinary fluids. Actually, we have obtained very simple forms of some exact solutions which either have already been obtained or are firstly obtained as limiting cases of our solution.

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AN INFORMATIONAL STUDY FOR THE $M/M/1/K$ QUEUEING SYSTEM

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Abstract

In this paper we will study the Shannon entropy, the Onicescu informational energy and the Fisher information for the queueing system $M/M/1/K$, for which the inter-arrival and service time are exponential random variables and the number of customers in the system is limited by K.
The considered random variables for which we study the above informational measures are the number of units in the system and the number of units in the queue. We will study the properties of the above values in function of $\rho = \frac{\lambda}{\mu}$ and K.

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MULTI-TIME VECTOR VARIATIONAL PROBLEMS

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Abstract

In this paper we establish necessary optimality conditions for multi-time scalar and vector variational problems.
Special necesary optimality conditons for variational problems defined on a generalized parallelepiped are proved using a projection method.
For some of the variational problems, above studied, we also establish sufficient optimality conditions, using the notion of invexity. Isoperimetric problems for scalar and vector variational problems are solved.

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ON THE SPECTRAL SEQUENCE ASSOCIATED TO THE KO-THEORY OF A COMPLEX STIEFEL MANIFOLD

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Abstract

Let W(n, k) be the complex Stiefel manifold (n>k), i.e. the variety of orthonormal k-frames from C^n. The purpose of this note is to study the Atiyah-Hirzebruch spectral sequence associated to the Real K-Theory of W(n, k).
Because the complex Stiefel manifold is a finite CW-complex and his cohomology H* (W(n, k); Z) has no torsion, the differentials $d_r^{p,q}$ of the above spectral sequence vanish for $r \geq 2$ and particular values of q.
I shall only sketch the proof of the main result of the paper; on the other hand, I shall assume the reader has some acquaintance with topological K-theory.

SOME FIXED-POINT RESULTS FOR FAMILIES OF MULTIMAPS IN THE FINITE DIMENSIONAL TOPOLOGICAL VECTOR SPACES SETTING AND THEIR APPLICATIONS

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Abstract

In this paper we apply some fixed-point results to deduce new fixed-point theorems and then to establish new coincidence theorems for families of compact multimaps in the finite dimensional topological vector spaces setting.
Also we apply some of the previous results to obtain existence theorems of equilibria problem (respectively of maximal element problem) for generalized abstract economies with two companies (respectively for qualitative games with two teams).

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CLOSURE OPERATORS, GALOIS CONNECTIONS AND DEDEKIND COMPLETION OF AN ORDERED SET

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Abstract

A map ϕ defined on an ordered set *P* is called a *closure* operator if it is idempotent, order preserving and extensive (i.e, $p \le \phi(p)$, for all $p \in P$). If ϕ is anti-extensive (i.e, $\phi(p) \le p$, for all $p \in P$), then ϕ is called an *anti-closure* operator (or an interior operator).

If *P* and *Q* are two ordered sets and $\pi^* : P \to Q$ and $\pi_* : Q \to P$ are two maps, then the pair (π^*, π_*) is called a *Galois connection* between *P* and *Q* if for all $p \in P$ and for all $q \in Q$ the following equivalence holds: $\pi^*(p) \le q \Leftrightarrow p \le \pi^*(q)$.

In the first part of the paper I show the relations between closure operators and Galois connections and their use to construct the Dedekind completion of an ordered set.

If *X* is a topological space and $B_{loc}(X)$ denotes the Dedekind complete lattice of all locally bounded real-valued functions defined on *X*, then the map $S : B_{loc}(X) \to B_{loc}(X)$ defined by $S(f)(x) = \inf\{\sup\{f(y) : y \in V\} : V \in N_x\}$, $x \in X$, is a closure operator, and the map $I : B_{loc}(X) \to B_{loc}(X)$ defined by $I(f)(x) = \sup\{\inf\{f(y) : y \in V\} : V \in N_x\}$, $x \in X$ is an anti-closure operator (here N_x denotes the set of all neighborhoods of x). These operators are called *Baire operators*: *S* is the upper Baire operator and *I* is the lower Baire operator.

In the second part of the paper I discuss how Baire operators are used in the above construction to obtain the Dedekind completion of $C(X)$ (the set of all continuous real-valued functions on *X*) with interval-valued functions.

A NOTE ABOUT SPLINE INTERPOLATION USING MATHCAD

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Abstract

It is well kwon that in order to obtain a cubic spline function we have to determine $4n$ unknowns and we only have $4n-2$ conditions. To determine a unique cubic spline function it is necessary to add two additional boundary conditions. In literature [1, 2, 3] several conditions are known that lead to obtaining diverse spline functions which differ among themselves only near the endpoints.
Mathcad uses for interpolation three internal functions: lspline, pspline and cspline. In the Mathcad Help [4] these functions have a very short presentation, which contains no indications about the boundary conditions used by that function.
In this paper I show what are the boundary conditions used by every of these Mathcad functions. For this I construct some spline functions in two ways: first with the Mathcad functions lspline, pspline and cspline and secondly directly using an explicit algorithm. Then I compare their graphs and their values in some points.

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MORITA EQUIVALENCE AND MORITA CONTEXT ASSOCIATED TO VON NEUMANN REGULAR RINGS

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Abstract

A ring R is called von Neumann regular if for any $r \in R$ there exists $s \in R$ such that $r = rsr$. Zelmanowitz generalizes this concept to modules in [4]: a left module M over the ring R is called regular if for each $m \in M$ there exists $g \in Hom_R(M,R)$ such that $g(m)m = m$. In paper [2] it was defined the concept of a regular object with respect to another object (or relative regular object) in an arbitrary category, which extends the notion of regular module. In the fist part of this paper we use the concept of relative regular module in order to give a new proof of a classical result: the von Neumann regular property of a ring is Morita invariant. In the second part we investigate the von Neumann regularity of the ring associated to a Morita context.

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TIME-VARYING SPECTRAL ANALYSIS OF NONSTATIONARY NON-GAUSSIAN SIGNALS

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Abstract

The higher-order time-frequency distributions are extensions of the quadratic time-frequency transforms in terms of instantaneous higher-order moments. These distributions, including third-order and fourth-order Wigner moment spectra, are considered for the time-varying spectral analysis of non-stationary non-Gaussian signals. Simulated and recorded signals are analyzed using higher-order multi-resolution methods. Performances of the time-varying second-order spectrum and sliced Wigner bispectrum and trispectrum are compared.

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TIME-FREQUENCY ANALYSIS OF SIMULATED EARTHQUAKE MOTIONS

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Abstract

The affine quadratic time-frequency distributions are considered for energetic characterization of the simulated earthquake accelerograms. The ground motion time histories compatible with the prescribed response spectrum can be simulated by different methods based on stochastic models. The bi-dimensional time-frequency representations of energy are used to evaluate the nonstationarity characteristics and the destructive potential of artificial strong ground motions.

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GENETIC ALGORITHMS IN HYDROLOGY

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Abstract

The genetic algorithm (GA) is a nonconventional optimum search technique and they are using an approach similar to the biological processes of natural selection and evolution. Unlike traditional methods, the genetic algorithm uses the objective itself, not the derivative information. Various statistical approaches require restrictive assumptions such as stationarity, homogeneity and normal probability distribution of the hydrological variables

concerned. The GAs do not require any of these assumptions in their applications. The aim of this paper is to detail the technique of GAs and how they can be utilized to optimize solutions to engineering issues. Many problems related to water resources require the optimization of the solution with the help of GAs.

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ON A FAMILY OF THIRD-ORDER DISPERSIVE EQUATIONS

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Abstract

Since 1955, there has been a strong interest in analyzing the properties of the equation [1]

$$z_t = B_x^2(z, z_x, z_{xx}), \quad B = -c_0 z + c_1 z^2 + c_2 z_x^2 + c_3 z z_{xx} - \gamma z_{xx}, \tag{1}$$

where $z(x,t)$ is the physical displacement, bounded for $x \to \infty$ and $t \to \infty$, c_i, $i = 0,1,...,3$ are parameters, and γ a dimensionless material constant. The index represents the differentiation with respect to the specified variable. Equation (1) arises in various physical context, such as the shallow water waves ($c_2 = c_3 = -c_1/2 = 1, c_0 = 0$), the blood motion through arteries ($c_1 = -3/2$, $c_2 = 1/2$, $c_3 = 1$) , or mechanical vibrations in a compressible elastic rod. Making the transformation $z(x,t) = \varphi(x - vt) = \varphi(\xi)$, eq.(1) can be reduced to a nonlinear equation

$$-v\varphi' = B'^2, \quad B = -c_0\varphi + c_1\varphi^2 + c_2\varphi'^2 + c_3\varphi\varphi'' - \gamma\varphi'', \tag{2}$$

where prime is the derivative with respect to ξ, and v is the wave speed. In this paper we solve (2), by taking the solution of the form [2]

$$\varphi(\tau) = \frac{\lambda\varphi^{(1)}(\tau)}{1 + \rho\varphi^{(1)}(\tau)}, \tag{3}$$

where λ and ρ depend on γ, and

$$\varphi^{(1)}(\tau) = \frac{3}{2}\varphi_0\sqrt{c_1/\gamma}\,\mathrm{sech}^2\left[(v - c_0)\sqrt{\frac{1}{6\gamma}\varphi_0\,|c_1 - kc_3|\,\tau}\right],. \tag{4}$$

.is a solution of the linearized version of (2), with $k = c_2 / c_3 \neq 0$ and $\mathrm{d}\xi = (\gamma - c_3\varphi)\mathrm{d}\tau$. Though the solution (3) is non-dispersive, we show that the waves described by (4) are dispersive. Finally, some profiles of solution are graphically represented.

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BAYESIAN NETWORKS IN MEDICAL DECISION ACT

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Abstract

The goal of this work is to present an application of Bayesian networks than medical decision support.
Bayesian networks with their associated methods are especially suited for capturing and reasoning with uncertainty. They have been around in biomedicine and health-care for more than a decade now and have become increasingly popular for handling the uncertain knowledge involved in establishing diagnoses of disease, in selecting optimal treatment alternatives, and predicting treatment outcome in various different areas. We propose an interactive Bayesian-network and decision-theoretic systems which intend to assist medical doctors in diagnosing prostate cancer, predicting likely outcome and selecting appropriate treatment. The main reason for this is that it is still a major undertaking to develop systems for problems of the complexity in monitoring prostate cancer.

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NUMERICALLY INTERPOLATIONS FOR INVERSE ABEL TRANSFORM INTEGRAL EQUATION - APPLICATION TO PLASMA SPECTROSCOPY

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Abstract

The purpose of this paper is to determine the radial distribution of the emission coefficient from the measured intensity distribution emitted by an extended source of radiation, particularly a plasma source. The source is assumed to be optically thin and axially symmetrical. This problem is solved by inverting Abel's integral equation. Abel's integral equation is frequently applied in the study of extended radiation sources with cylindrical symmetry. A measurement of the transverse distribution $I(y)$ of the intensity emitted perpendicularly to the source axis allows the calculation of the emission coefficient radial distribution $F(r)$. If the source is optically thin, the intensity $I(y)$ is connected to the emission coefficient by the formula:

$$I(y) = \int_{-x}^{x} F(r)dx$$

F(r) can be deduced from *I(y)* by the inverse formula:

$$F(r) = -\frac{1}{\pi}\int_{y}^{R} \frac{dI(y)}{dy} \frac{1}{\sqrt{r^2 - y^2}} dy$$

known as Abel's integral equation. A smoothing procedure is made on the experimental curve in order to attenuate the random errors before computing the derivative. The integral is calculated using a polynomial of second degree for the approximation of $dI(y)/dy$ in a small interval on the right of the discontinuity point, the other part is calculated using an approximate numerical method given by the function *intsplin* of the Scilab program.

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APPLICATIONS OF GEOMETRIC PROGRAMMING

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Abstract

This paper represents an introduction in the theory of geometric programming, based on practical examples given by science and technique.
The theoretical part is accompanied by complete solved problems and by applications from different domains. All the results are given with a minimum formality, but with respectful of mathematical strictness.

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NEW ASPECTS OF THE RENORMALIZATION PROBLEM FOR DISCRETE GENERALIZED HYPERGROUPS

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Abstract

In the present paper we study, for the discrete Hahn polynomials on [0, N-1], the discrete generalized hypergroup (K, A_0), where $K = \{p_0, p_1,.., p_{n-1}\}$ is the finite sequence of normalized Hahn polynomials $p_i(x) = \dfrac{h_i^{\alpha,\beta}(x,N)}{d_i}$ (the notations are those from [ ]) and A_0 is the *-algebra linearly generated by K (see [ ], [ ]). We consider the set $S = \{X_0, X_1,.., X_{n-1}\}$ of characters on K. To each character X_j there corresponds a

renormalization generalized hypergroup (K_j, A_0) and we compare the different characteristics of the pairs (K_j, A_0). Moreover, we study sufficient conditions for the positivity of the structure constants associated to these generalized hypergroups, along with the associated Fourier transform.

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PROJECTIVE MATRIX SPACE

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Abstract

In the first part of the paper, we attach a (n-1)-matrix, denoted A_P, to any n-stochastic matrix A, A_P having projective equivalent eigenvalues with A. Thus we can define an application $\phi : M^{st}{}_{n\times n} \to M_{(n-1)\times(n-1)}$ given by $\phi(A) = A_P$. The resulted space is the projective space. This construction holds true also for A a generalized n-stochastic matrix.
In the second part, we consider A an arbitrary n-matrix, with $s_1 \neq 0$, where s_1 is the eigenvalues sum. Using our previous results [3], [4], we can attach to A a generalized n-stochastic matrix, A_S, having same eigenvalues as A. Following the previous construction, we can now define the projective (n-1)-matrix of A.

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A REPRESENTATION OF THE ALGEBRA OF ENTIRE FUNCTIONS

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Abstract

Let *K* be a valued field with respect to a nontrivial absolute value $|\ |$. We denote by

$$H(K) = \left\{ f = \sum_{k=0}^{\infty} a_k x^k : a_k \in K \text{ which converges for every } x \in K \right\}$$

the *K*-algebra of entire functions with coefficients in *K*. Then it follows easily that $f \in H(K)$ if and only if $\lim_{k\to\infty} \sqrt[k]{|a_k|} = 0$. If $S = \{\alpha_k\}_{k\geq 1}$ is a sequence of elements from *K*, consider the polynomials

$$u_0 = 1,\ u_k = \prod_{j=1}^{k} (X - \alpha_j),\ k \geq 1.$$

We put

$$H_S(K) = \left\{ g = \sum_{k=0}^{\infty} b_k u_k(x) : b_k \in K \text{ which converges absolutely for every } x \in K \right\}$$

Then we prove that, for suitable sequences *S*, $H_S(K)$ is a *K*-algebra isomorphic to *H*(*K*). Finally some applications of this result are given.

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USING THE EXTERNAL DATABASES

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Abstract

The external databases are some files which stock the information. We can create, edit or recover an immense volume of databases when we shall use the external databases for data organization and storage.
AutoCAD offers the tools which allow us to work with some files that contain the external databases:
we can open a database for data viewing or editing,
we can bind the database records with the AutoCAD objects (lines, circles, and so on),
we can execute the queries in order to recover a data subset based on some criterions.
AutoCAD is designed such that to assure the access to the external database files without to use the application of database creation.
AutoCAD recognizes the databases which where created with the following Database Management Systems: Access, dBase, Excel, Oracle, Paradox, Visual FoxPro. We shall construct an Access database in our paper.

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CALIBRATION OF GROUNDWATER MODELS USING PARTICLE SWARM OPTIMIZATION

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Abstract

Particle swarm optimization (PSO) is a population based stochastic optimization technique developed by Eberhart and Kennedy in 1995, inspired by social behavior of bird flocking or fish schooling. PSO shares many similarities with evolutionary computation techniques such as Genetic Algorithms (GA). The system is initialized with a population of random solutions and searches for optima by updating generations.

In a swarm of insects or a school of fish, if one sees a desirable path to go (for food) the rest of the swarm will be able to follow quickly even if they are on the opposite side of the swarm. On the other hand, in order to facilitate the exploration of the search space, each particle must have a certain level of randomness in their movement. This is a manifestation of the basic exploration-exploitation tradeoff that occurs in any search problem.

This is modeled by particles in multidimensional space $\Re^m$ that have a position and a velocity. These particles are flying through hyperspace and have two essential reasoning capabilities: their memory of their own best position and knowledge of the swarm's best, "best" simply meaning the position with the smallest value of the objective function. Members of a swarm communicate good positions to each other and adjust their own position and velocity based on these good positions.

The method was used to find the optimal parameters for a synthetic aquifer, the objective function beeing the sum of squared differences between observed and computed heads.

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GENETIC PROGRAMMING AND APPLICATIONS

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Abstract

By using some basic principles of nature, it is possible to develop relevant numerical methods for a wide area of mathematical models. Using informatics, as well as mathematics, genetic programming is a nonstandard but powerful tool, in the following framework: $\inf_{x\in P} f(x)$, where stands for an abstract set, called population.

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ON THE GEOMETRY OF GENERALIZED ORLICZ-SOBOLEV SPACES

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Abstract

In this paper, we bring a contribution to the study of the geometry of generalized Orlicz-Sobolev spaces $W_0^m L_M(\Omega)$, where M is a Musielak-Orlicz function. The space $W_0^m L_M(\Omega)$ is endowed with the norm $\|u\| = \left\|\sqrt{T[u,u]}\right\|_{(M)}$, $T[u,v]$ being a nonnegative symmetric bilinear form on $W_0^m L_M(\Omega)$, involving only the generalized derivatives of order m of the functions $u, v \in W_0^m L_M(\Omega)$.

VALUE AT RISK (VAR)

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Abstract

Value at Risk (VaR) is a widely used measure of the risk of loss on a specific portfolio of financial assets. For a given portfolio, probability and time horizon, VaR is defined as a threshold value such that the probability that the mark-to-market loss on the portfolio over the given time horizon exceeds this value (assuming normal markets and no trading in the portfolio) is the given probability level.

In this paper we present a relative risk-value model and derive a relative measure of risk with positive outcomes.

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DYNAMICS IN THE LENNARD-JONES MODEL

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Abstract

We depict the global flow of the two-body problem associated to the Lennard-Jones potential. To this end we use both qualitative analysis and numerical approach. The phase space, described in McGehee coordinates, is foliated according to the energy level (negative, zero, positive) and to the angular momentum. In this way we obtain phase portraits which exhibit a large variety of orbits and can be interpreted in terms of physical trajectories.

THE RESONANCE EFFECT ON THE SEISMIC RESPONSE OF STRUCTURES

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Abstract

This paper is studying the resonance effect on the seismic response of structures. The numerical simulation is performed on multi-story building modeled as a three blocks structure. On this structural model with three levels is analyzed the dynamic response, to the same earthquake motion, of some structures with different stiffness. The response intensity is evaluated by a global index considering the relative displacements between levels and the absolute accelerations. The results obtained by numerical simulation show clearly that in buildings under the influence of earthquakes with dominant spectral components could show up dynamic response intensifications because of the resonance phenomenon. The conclusion is important considering the fact that this is a question many times ignored by the reason that resonance phenomenon can not develop because of the relatively short time of the seismic motion.

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ON THE BEHAVIOR OF BEAMS WITH EXTERNAL AUXETIC PATCHES

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Abstract

The ability of tailoring the best behavior of beams at vibration consists in a qualitative and quantitative understanding of the damping properties. One way to manipulate the eigenfrequencies of the beams is to vary its damping capacity.

The paper discusses the behavior of beams with external auxetic damping patches. The damping force is modeled by using the nonlocal theory [1]. The governing equation of motion for a 1D linear damped continuous dynamic system may be expressed as [2]

$$Lu(x,t) = 0,\ \ x \in \Omega,\ \ t \in [0,T], \tag{1}$$

where $u(x,t)$ is the displacement vector, x is the spatial variable, t is time, and L is the nonlocal operator defined by

$$Lu(x,t) = \rho(x)\frac{\partial^2}{\partial t^2}u(x,t) + M\frac{\partial}{\partial t}u(x,t), \tag{2}$$

where $\rho(x)$ is the distributed mass density. The operator M is defined as

$$M\frac{\partial}{\partial t}u(x,t) = \int_{\Omega}\int_{0}^{t} C(x,\xi,t-\tau)\frac{\partial}{\partial t}u(\xi,\tau)\mathrm{d}\tau\mathrm{d}\xi, \tag{3}$$

with $C(x,\xi,t-\tau)$ the kernel function for external damping which is only dependent on the displacement. If the damping kernel functions are assumed to be separable in space and time, we can write $C(x,\xi,t-\tau)$ in a general form

$$C(x,\xi,t-\tau) = H(x)c(x-\xi)g(t-\tau). \tag{4}$$

The expression (4) represents the general form of viscoelastic damping model. The function $H(x)$ denotes the presence of nonlocal damping. We have $H(x) = H_0$ (constant) if x is within the patch., and $H(x) = 0$ otherwise. The performance with respect to eigenvalues is discussed next, in order to avoid resonance. .

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APPROXIMATE INERTIAL MANIFOLDS AND ABSORBING DOMAINS FOR LOTKA-VOLTERRA MODEL

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Abstract

Inertial manifolds are tools for describing the large time behaviour of p.d.e and o.de. They are used to reduce the infinite-dimensional case to a finite-dimensional case or from finite-dimensional to another finite-dimensional space, but with lower dimension. Each phase trajectorie is approximated with one from the inertial manifold. In this paper, the construction of the approximate inertial manifolds for one prey-predator model, Lotka-Volterra, is based on the determination of the absorbing domains using the phase portraits for a few choices of parameters satisfying the Jolly-Rosa-Temam hypothesis. Errors of approximation are computed using the same algorithm.

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TAYLOR MONOMIALS ON TIME SCALES AND SOME OF ITS APPLICATIONS

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Abstract

The theory of time scales was introduced by Stefan Hilger in his 1988 PhD thesis in order to unify continuous and discrete analysis.

In this paper we will define the “Taylor Monomials” for a general time scale and we will calculate this monomials for different time scales. Also, a Taylor’s formula for time scales is presented, which in helpful of boundary value problems.

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SOME REMARKS ON THE RADON TRANSFORM AND ITS NUMERICAL IMPLEMENTATIONS

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Abstract

The Radon transform is the basic tool of the computerized tomography. In the sequel we introduce this transform, review its properties and compare some numerical implementations for its inversion.

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ON KRASNER'S LEMMA IN P-ADIC COMPLEX NUMBERS

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Abstract

Let a and b be two algebraic elements over the p-adic number field Q_p.The classical Krasner's Lemma says that if the p-adic distance between a and b is less than the p-adic distance from a up to the nearest of its conjugates (over Q_p), then a is a polynomial in b with coefficients in Q_p, i.e. $Q_p(a)$ is contained in $Q_p(b)$. The converse of this statement is clearly not true. This is because the distance hypothesis is too strong. In this paper we generalize this form of Krasner's Lemma for arbitrary elements of C_p (even transcendental) and, at the same time, we give an equivalence of two statements instead of one implication only.

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BASE ISOLATION FOR EARTHQUAKE BUILDING PROTECTION WITH SEMI-ACTIVE CONTROL

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Abstract

A study has been made regarding the use a semi-active friction actuator, herein reffered to as a damper, to control the seismic response of a building modeled as three masses. The response intensity is evaluated through a global index considering the relative displacements between levels and the absolute accelerations. The results obtained by numerical simulation show that base isolation reduces the peak accelerations, the base displacement and the inter-storey drift up to 40% and thereby the transmissibility of ground acceleration to the protected structure.

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THE DUGUE PROBLEM IN THE DISCRET CASE

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Abstract

This problem, proposed by Dugue in 1939, consists in determining the independent random variables X, Y with the property: $\exists p \in (0,1)$ such that

$$\varphi_{X+Y} = p\varphi_X + (1-p)\varphi_Y$$

where φ_X is the characteristic function of the variable X. We name this kind of variables *p-conjugated*.

We present a solution of the problem in the hypothesis X, Y have discrete values, $X \geq 0, Y \leq 0$ a.s.

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THERMAL AND THERMOELASTIC WAVES IN CATTANEO'S TYPE BARS

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Abstract

In this paper we analyse a thermal shock problem in semiinfinite bar of Cattaneo's type. The Cataneo type rigid bar and thermoelastic bar models are presented, which take into consideration the lost of heat through the bar's lateral surface. The hyperbolic character of the field equations is also discussed.

The paper analyses the propagation of the Hadamard's type surfaces of discontinuity, of order 0 and 1, for the fields involved in the case of thermal and thermoelastic models. The analysis of the shock waves is based on the concept of weak (generalised) solution introduced by Courant and Hilbert. Out of the cinematic and dynamic relations of compatibility, the equations of transport are deduced. By integrating these equations, we can see how the jumps of the involved fields vary in space and time.

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OPTIMIZING THE SUM OF LINEAR FRACTIONAL FUNCTIONS

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Abstract

The global optimization of the sum of linear fractional functions has attracted the interest of researchers and practitioners for a number of years. Given a nonempty, compact convex set, the method determines a function that is the sum of linear fractional functions and attains a global minimum over the set at a point that can be found by convex programming and univariate search. In this paper we will present a method for constructing test problems of the form of problem (PN). Generally, the function will have also local minima over the set that are not global minima.

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DÜFFING'S EQUATION TREATED BY LEM

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Abstract.

LEM – the linear equivalence method – was previously introduced by the author to the purpose of finding and studying, both numerically and qualitatively, the solutions of non-linear dynamical systems depending on parameters in a classical linear frame. LEM is applied

here to Düffing's equation, that can be interpreted as a forced oscillator with a spring whose restoring force is of polynomial type. The normal LEM solutions in the case of a damped oscillator of Ueda type are established, also testing them numerically by using the Runge-Kutta method. It is also noted that the free Düffing oscillator is similar to a certain intrinsic equation, previously found and studied, emphasized as the mathematical core of several distinct models, corresponding to physical models belonging to both solid and plasma state; the LEM solution of the intrinsic equation could then be conveniently adapted to this case.

VON MISES DISTRIBUTION AND APPLICATIONS

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Abstract.

In probability theory and statistics, the von Mises distribution (also known as the circular normal distribution) is a continuous probability distribution on the circle. It may be thought of as a circular analogue of the normal distribution (another is the wrapped normal distribution). In this paper we present von Mises distribution and how is used in applications of directional statistics, where a distribution of angles is found which is the result of the addition of many small independent angular deviations, such as target sensing, or grain orientation in a granular material. The von Mises distribution is a special case of the von Mises-Fisher distribution on the N-dimensional sphere.

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ABOUT DOUBLE ITERATED CROSSED PRODUCT OF A CONTINUOUS TRACE C^*-ALGEBRA BY AN ABELIAN GROUP

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Abstract.

In this paper, a *dynamical system* is a triple (A,G,α) consisting of a C^*-algebra *A*, a locally compact, an abelian group G and a continuos homomorphism $\alpha : G \to Aut(A)$, where $Aut(A)$ is the notation for the group of automorphismes of A. Given such a dynamical system will denote by $A \times_\alpha G$ the C^*-algebra crossed product asociated to system (A,G,α). Considering $\hat{G}$, the dual group of G, we get a homomorphism $\hat{\alpha} : \hat{G} \to Aut(A \times_\alpha G)$, such that triple $(A \times_\alpha G, \hat{G}, \hat{\alpha})$ will be a dynamical system, and have sense to form the double iterated crossed product $(A \times_\alpha G) \times_{\hat{\alpha}} \hat{G}$. The Pontryagin duality theorem allows us to identify G with the dual of $\hat{G}$, and the result of Takai shows that $(A \times_\alpha G) \times_{\hat{\alpha}} \hat{G}$ is isomorphic to $A \otimes K(L^2(G))$. By a spectrum of a C^*-algebra, $\hat{A}$, we mean the set of unitary equivalence classes of irreducible representations of *A* together with Jacobson topology, and a C^*-algebra *A* is a continuous trace C^*-algebra if it has Hausdorff spectrum and for every $\pi_0 \in \hat{A}$, there is a positive element *a* of *A* such that $\pi(a)$ is a rank-one projection, for π in a neighbourhood of π_0.

The main result of this paper is:

Theorem If (A,G,α) is a dynamical system, such that C^*-algebra *A* is a continuous trace C^*-algebra, then the spectrum of the double iterated crossed product $(A \times_\alpha G) \times_{\hat{\alpha}} \hat{G}$ is isomorphic to spectrum of *A*.

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SOME REMARKS ON FIXED POINT THEORY ON MULTIVALUED OPERATORS IN ORDERED SPACES

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Abstract.

The purpose of this paper is to present some fixed point results in ordered spaces for multivalued mappings.
Let X be a complet vector lattice with a vector metric. Then we denote:
(X)={Y/ is a subset of X} ; P(X)={$Y \in$ □(X) / Y is a nonempty set}
If $T : X \to$ P(X) is a multivalued operator from X to X, then: $T^1 := T$, $T^2 := T \circ T$,…, $T^{n+1} := T^n \circ T$, Fix(T)={ $x \in X$ / $x \in T(x)$ } the fixed point set of T.
We consider the following single valued operator generated by T:
$\hat{T}$: P(X)→P(X), $Y \to T(Y) := \bigcup_{y \in Y} T(y)$.
The operator $\hat{T}$ is called the fractal operator coresponding to T. We mention that an operator $T : X \to X$ is called Picard operator if Fix(T)={x*} and $T^n(x) \to x^*$ as $n \to \infty$.
<u>*Theorem*:</u> Let X be a complete vector lattice and $Y \subset P_{cl}(X)$ be such that $x \in X$ implies $\{x\} \in Y$. Let $T : X \to Y$ be an u.s.c. multivalued operator such that $W \in Y$ implies $T(W) \in Y$. We suppose that $\hat{T} : Y \to Y$ is an Picard operator and denote by W_T^* the unique fixed point of $\hat{T}$. Then:
(i) Fix(T) $\subset W_T^*$ and $W_T^* = \bigcup_{n \in N} T^n(x)$ for all $x \in$Fix(T);
(ii) If T(Fix(T))= Fix(T) and Fix(T)$\in Y$, then $T^n(x) \to$ Fix(T) as $n \to \infty$ for all $x \in X$.

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THE GEOMETRY OF THE SPHERE WITH MATHCAD

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Abstract.

Mathcad readily meets the criteria as an accessible and dynamic tool, and provides the ideal solution for students to understand the symbolism that is used to make the abstract concepts geometry (points, lines, planes and surfaces) observable and measurable.
In this paper we focus on the specific geometry on the surface of the sphere which is more intuitive and we present an example in which we provide a Mathcad worksheet for some of the basic concepts of this geometry.

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ABOUT ORTHOGONALITY IN HILBERT C*-MODULES

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Abstract.

This paper is dedicated to the orthogonality of the submodules of Hilbert C*-modules.
In section 2 we present Lance's example ([3]) that emphasizes the difference between Hilbert spaces and Hilbert C*-modules, i.e. the fact that every closed submodule has not necessarily an orthogonal complement.

In Section 3 we give Magajna's characterization of all C*-algebras B such that all closed submodules in any Hilbert module over B are orthogonally complemented ([4]). He proved that if B is a C*-algebra which admits a full Hilbert B-module E such that every closed right submodule of E is orthogonally complemented, then B is necessarily isomorphic to a C*-subalgebra of the algebra of compact operators on a Hilbert space.

In Section 4 we remind Schweizer's theorem ([9]) that has the same conclusion, but with a weaker assumption that every closed submodule E_0 is orthogonally closed in the sense $E_0 = \left(E_0^{\perp}\right)^{\perp}$.

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THE HOMOTOPY ANALYSIS METHOD AND DUFFING EQUATION

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Abstract.

In this work, we consider the Duffing equation [1,2] with a periodic perturbating factor. The periodic solution is obtained by applying the homotopy analysis method [3]. This solution is in good agreement with those calculated by computational methods. This highlight the homotopy analysis method as a an useful tool to solve nonlinear differential equations.

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